Fluids Quiz 1 Solution - Problem 3 S'08

Fourier series representation: $\Gamma = 2V_\infty b\left[A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + \dots\right]$

This case : $\Gamma = V_\infty b\left[0.05 \sin\theta \quad - 0.005 \sin 3\theta\right]$

$\therefore A_1 = 0.025 \,, \quad A_3 = -0.0025$

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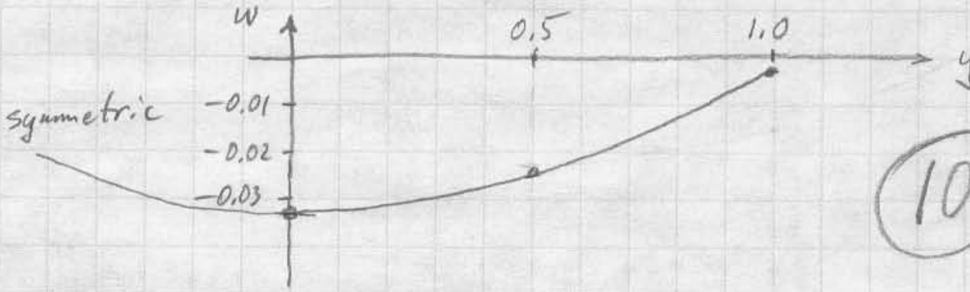
a) $\boxed{L = \frac{\pi}{2}\rho V_\infty^2 b^2 A_1 = \frac{\pi}{2} 1\cdot 1^2 \cdot 2^2 \cdot 0.025 = 0.1571}$

$\boxed{D_i = \frac{\pi}{2}\rho V_\infty^2 b^2 A_1^2\left[1 + 3\left(\frac{A_3}{A_1}\right)^2\right] = \frac{\pi}{2}\cdot 1 \cdot 1^2 \cdot 2^2 \cdot 0.025^2\left[1 + 3\left(\frac{0.0025}{0.025}\right)^2\right] = 0.00404}$

b) $w = -\alpha_i V_\infty = -V_\infty \sum n A_n \frac{\sin n\theta}{\sin\theta} = -V_\infty\left[A_1 + 3A_3\left(4\cos^2\theta - 1\right)\right]$

using $\cos\theta = \frac{2y}{b}$, $V_\infty = 1$, $b = 2$: $\boxed{\begin{aligned} w &= -\ 0.025 + 3\cdot 0.0025\left(4y^2 - 1\right) \\ &= -\ 0.0325 + 0.03\,y^2 \end{aligned}}$

y	w
0	-0.0325
0.5	-0.0250
1.0	-0.0025



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c) $\Gamma = \frac{1}{2} V_\infty c\, c_\ell = V_\infty b\left[0.05 \sin\theta - 0.005 \sin 3\theta\right]$

$c = \frac{2b}{c_\ell}\left[\quad '' \quad\right] = 4\left[0.05 \sin\theta - 0.005 \sin\theta\left[4\cos^2\theta - 1\right]\right]$

since $\sin\theta = \sqrt{1 - \left(\frac{2y}{b}\right)^2} = \sqrt{1 - y^2}$, $\sin 3\theta = \sin\theta\left[4\cos^2\theta - 1\right] = \sqrt{1-y^2}\left(4y^2 - 1\right)$

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$\boxed{c = \sqrt{1-y^2}\left[0.2 - 0.02\left(4y^2 - 1\right)\right] = \sqrt{1-y^2}\left[0.22 - 0.08\,y^2\right]}$

d) For this case $\alpha = 0$ is required, and $\alpha_i = \sum n A_n \frac{\sin n\theta}{\sin\theta} = A_1 + 3A_3\left(4\cos^2\theta - 1\right)$

$c_\ell = 2\pi\left[\alpha_{aero} + \cancel{\alpha} - \alpha_i\right]$

$\boxed{\begin{aligned} \alpha_{aero} &= \frac{c_\ell}{2\pi} + \alpha_i = \frac{1}{2\pi} + 0.025 + 3\cdot 0.0025\left(4y^2 - 1\right) \\ &= 0.1842 + 0.0075\left(4y^2 - 1\right) \\ &= 0.1761 + 0.03\,y^2 \end{aligned}}$

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